NASA TN D-97

7N-05
195540
16 P.

TECHNICAL NOTE

D-97

A NOTE ON THE CALCULATION OF HELICOPTER PERFORMANCE

AT HIGH TIP-SPEED RATIOS

By Alfred Gessow

Langley Research Center
Langley Field, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

WASHINGTON September 1959

1L

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TECHNICAL NOTE D-97

A NOTE ON THE CALCULATION OF HELICOPTER PERFORMANCE AT HIGH TIP-SPEED RATIOS

By Alfred Gessow

L
5
9
8

SUMMARY

A helicopter performance procedure is described and illustrated that accounts for the exact orientation and magnitude of the rotor resultant-force vector. The procedure utilizes existing rotor performance charts and is applicable to operation at high tip-speed ratios.

INTRODUCTION

When an existing rotor theory, such as the charts of reference 1, is applied to the calculation of helicopter performance, the assumption is normally made that either the rotor thrust is equal to the gross weight of the helicopter in level flight or the contribution to the rotor resultant force of the rotor force that acts normal to the axis of no feathering (the so-called H-force) can be ignored. These assumptions are reasonable in the present-day helicopter speed range (up to tip-speed ratios of about 0.3). In calculations of helicopter performance at high forward speeds, however, (at tip-speed ratios in the neighborhood of 0.5, for example) where the rotor may be on the verge of, or in, the stall and compressibility regions, the calculated power required may be significantly affected if the exact orientation and magnitude of the rotor resultant force are not considered in the calculation of the rotor thrust coefficient.

The purpose of this paper is to set forth a relatively convenient means for using published rotor theories (such as ref. 1) for the calculation of helicopter performance which eliminates small angle assumptions in regard to the magnitude and direction of the rotor force vector. The method will be illustrated by an example.

SYMBOLS

a' — longitudinal displacement of resultant-force vector from axis of no feathering, deg

B — tip-loss factor; blade elements outboard of radius BR are assumed to have profile drag but no lift

b — number of blades per rotor

C_H — inplane drag coefficient, $\frac{H}{\pi R^2 \rho (\Omega R)^2}$

C_P — rotor-shaft power coefficient, $\frac{P}{\pi R^2 \rho (\Omega R)^3}$

C_T — rotor thrust coefficient, $\frac{T}{\pi R^2 \rho (\Omega R)^2}$

C_W — weight coefficient, $\frac{W}{\pi R^2 \rho (\Omega R)^2}$

c — blade-section chord, ft

c_e — equivalent blade chord (on thrust basis), $\frac{\int_0^R cr^2 dr}{\int_0^R r^2 dr}$, ft

D_p — parasite-drag force, lb

f — parasite-drag area, $\frac{D_p}{1/2 \rho V^2}$, sq ft

F_R — resultant rotor force in longitudinal plane of symmetry, lb

H — component of rotor resultant force in longitudinal plane of symmetry and perpendicular to axis of no feathering, lb

P — rotor-shaft power, ft-lb/sec

r — radial distance from center of rotation to blade element, ft

L
5
9
8

R	rotor radius measured from center of rotation, ft
T	component of rotor resultant force along axis of no feathering in longitudinal plane of symmetry, lb
v	induced velocity at rotor (always positive), fps
V	true airspeed of helicopter along flight path, fps
W	helicopter gross weight, lb
α	rotor angle of attack; angle between flight path and plane perpendicular to axis of no feathering, positive when axis is pointing rearward, deg
γ	flight-path angle (positive in climb, negative in glide), deg
$\theta_{0.75}$	blade-section pitch angle at 0.75 radius; angle between line of zero lift of blade section and plane perpendicular to axis of no feathering, deg
θ_1	difference between blade-root and blade-tip pitch angles, positive when tip angle is larger, deg
λ	inflow ratio, $\frac{V \sin \alpha - v}{\Omega R}$
μ	tip-speed ratio, $\frac{V \cos \alpha}{\Omega R}$
ρ	mass density of air, slugs/cu ft
σ	rotor solidity, $\frac{bc_e}{\pi R}$
Ω	rotor angular velocity, radians/sec
$\alpha_{(1.0)(270^\circ)}$	blade-element angle of attack at tip of blade at 270° azimuth position, measured from line of zero lift, deg
$\alpha_{(u_T=0.4)(270^\circ)}$	blade-element angle of attack at radius at which the tangential velocity equals 0.4 tip speed and at 270° azimuth position, deg

L
5
9
8

Subscripts:

c climb

i induced

o profile

p parasite

L
5
9
8

FUNDAMENTAL PERFORMANCE RELATIONS

The division of shaft power among the various sources can be written in coefficient form as

$$C_P = C_{P,o} + C_{P,i} + C_{P,p} + C_{P,c} \tag{1}$$

From figure 1, the equation for force equilibrium perpendicular to the flight path can be deduced as

$$C_T \frac{\cos(\alpha + a')}{\cos a'} = C_W \cos \gamma \tag{2}$$

Note that, in accordance with standard NASA rotor notation, positive α is defined for the case wherein the axis of no feathering (or rotor thrust) is inclined backward from the perpendicular to the flight path and that rotor thrust is defined as the component of the resultant rotor force vector acting along the axis of no feathering.

Additional relationships that are required are

$$C_{P,i} = \frac{{C_T}^2}{2B^2\left(\lambda^2 + \mu^2\right)^{1/2}} \tag{3}$$

$$C_{P,p} = \frac{1}{2} \frac{f}{\pi R^2} \left(\frac{\mu}{\cos \alpha}\right)^3 \tag{4}$$

$$C_{P,c} = C_W \mu \frac{\sin \gamma}{\cos \alpha} \tag{5}$$

$$\tan \alpha = \frac{\lambda}{\mu} + \frac{C_T}{2B^2\mu\left(\lambda^2 + \mu^2\right)^{1/2}} \tag{6}$$

$$\tan a' = \frac{1}{\mu}\left(\frac{C_{P,o}}{C_T} - \frac{C_P}{C_T} - \lambda\right) \tag{7}$$

Equations (3) to (6) are standard in the literature and can readily be derived. Equation (7), which is derived in the appendix, makes the direct calculation of the H-force unnecessary. (A similar equation, which contains some small-angle assumptions, may be found in ref. 2.) The power coefficient $C_{P,o}$ can be obtained from the charts of reference 1 by the methods discussed in that reference or from sources that account for stall and compressibility effects.

OUTLINE OF PERFORMANCE METHOD

The calculation of power required from the preceding relations requires the solution of a large number of nonlinear simultaneous equations that can best be accomplished by an iteration process. The procedure would be as follows for the known parameters B, W, σ, f, γ, Ω, R, ρ, and V:

(1) Assume that α, a', and $\lambda = 0$ and calculate C_T from equation (2).

(2) Calculate μ from its definition and $C_{P,i}$, $C_{P,p}$, and $C_{P,c}$ from equations (3), (4), and (5), respectively. With these values, $C_P - C_{P,o}$ can be obtained from equation (1).

(3) From profile-power charts (of ref. 1, for example) determine C_P, $C_{P,o}$, and $\theta_{0.75}$.

(4) By using this value of $\theta_{0.75}$ determine λ from plots of the variation of C_T with λ (of ref. 1, for example).

(5) Calculate α and a' from equations (6) and (7). This completes one iteration.

The process should be repeated until values of C_T, $C_{P,o}$, C_P, μ, α, and a' obtained from the terminal iteration agree with the values from the preceding iteration to within a predetermined percentage. Experience with the process indicates that the values vary with iteration number in a manner similar to that of a damped oscillation. In cases wherein the oscillation steadies down to constant amplitude, the amplitude is small and is within the accuracy of the calculations. Generally, six iterations or less are sufficient even in extreme cases, and the values are fairly well "zeroed-in" after the first couple of iterations. The process of convergence may be improved by plotting input values against output values and using the intersections of the curve formed by the points with the line of perfect agreement (that is, the 45° line) as the input value for the next iteration.

SAMPLE CALCULATIONS

L
5
9
8

The performance method outlined in the preceding section was applied (by using the charts of ref. 1) to an investigation of the effect of parasite drag on the shaft power required by a helicopter traveling in level flight at a high forward speed. The data are as follows: $W = 12{,}000$ lb; $V = 200$ knots (338 fps); $\sigma = 0.10$; $\Omega R = 700$ fps; disk loading $= 4.87$ lb/sq ft; $\theta_1 = -8^{\circ}$; and $\rho = 0.002378$. The shaft power is required for the following values of f: 0, 6, 12, 18, and 24 square feet.

The results of each iteration for the $f = 12$ square feet case, and of the final iteration for the other f values are given in tables I and II.

Power-required values are plotted as a function of parasite-drag area in figure 2. The curve labeled "Initial iteration" was obtained from the first iteration only and, because it contains small-angle assumptions in the calculation of μ and C_T, it represents the usual method of performance calculation. The curve labeled "Final iteration" reflects the marked reduction in power required resulting from the removal of the small-angle assumptions.

No stall or compressibility effects are represented in the calculations, since none are included in the charts of reference 1. The inclusion of such effects would be expected to accentuate the differences between the curves. An indication of the amount of rotor stall present at the various f values investigated is given by the $\alpha_{(1.0)(270^{\circ})}$ and $\alpha_{(u_T=0.4)(270^{\circ})}$ values listed in table II.

CONCLUDING REMARKS

A method of computing helicopter performance is described and illustrated that is more precise than existing procedures for conditions involving high forward speeds (tip-speed ratios above approximately 0.3). The method, which generally results in lower power requirements, uses published rotor charts and removes small-angle assumptions in regard to the magnitude and orientation of the rotor force vector and the calculation of tip-speed ratio.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., June 9, 1959.

APPENDIX

DERIVATION OF EQUATION FOR a'

As seen from figure 1, the longitudinal displacement of the resultant-force vector from the axis of no feathering is defined as a'. The displacement a' may be determined directly as $\tan^{-1} \frac{C_H}{C_T}$, where C_H can be calculated in the same manner as C_T. It is possible, however, to determine a' for a given flight condition from values of C_P, $C_{P,o}$, and C_T. The relationship can be derived as follows:

L598

From figure 1, the force equilibrium equation along the flight path is

$$F_R \sin(\alpha + a') + D_p + W \sin \gamma = 0 \tag{A1}$$

or

$$T \sin \frac{(\alpha + a')}{\cos a'} + D_p = -W \sin \gamma \tag{A2}$$

In nondimensional form, equation (A2) becomes

$$C_T \sin \frac{(\alpha + a')}{\cos a'} + C_{P,p} \frac{\cos \alpha}{\mu} = -C_W \sin \gamma \tag{A3}$$

Now

$$C_{P,c} = C_W \frac{\mu}{\cos \alpha} \sin \gamma \tag{A4}$$

Substituting equation (A4) into equation (A3) yields

$$C_T \frac{\sin(\alpha + a')}{\cos a'} + \frac{\cos \alpha}{\mu} \left(C_{P,p} + C_{P,c}\right) = 0 \tag{A5}$$

Expanding $\sin(\alpha + a')$ and substituting $C_P - C_{P,o} - C_{P,i}$ for $C_{P,p} + C_{P,c}$ in equation (A5), the equation becomes

$$C_T \frac{(\sin \alpha \cos a' + \cos \alpha \sin a')}{\cos a'} + \frac{\cos \alpha}{\mu} \left(C_P - C_{P,o} - C_{P,i}\right) = 0 \tag{A6}$$

which, after simplifying, reduces to

$$\tan\alpha + \tan a' + \frac{1}{\mu C_T}\left(C_P - C_{P,o} - C_{P,i}\right) = 0 \tag{A7}$$

Now

$$\tan\alpha = \frac{\lambda}{\mu} + \frac{C_T}{2B^2\mu\left(\mu^2 + \lambda^2\right)^{1/2}} \tag{A8}$$

and

$$C_{P,i} = \frac{C_T^2}{2B^2\left(\lambda^2 + \mu^2\right)^{1/2}} \tag{A9}$$

Substituting equations (A8) and (A9) into equation (A7) and simplifying, yields

$$\tan a' = \frac{1}{\mu}\left(\frac{C_{P,o}}{C_T} - \frac{C_P}{C_T} - \lambda\right) \tag{A10}$$

When calculating a' from equation (A10), as many significant figures should be used for $C_{P,o}$, C_P, and C_T as are available, inasmuch as small differences between numbers are involved.

REFERENCES

1. Gessow, Alfred, and Tapscott, Robert J.: Charts for Estimating Performance of High-Performance Helicopters. NACA Rep. 1266, 1956. (Supersedes NACA TN 3323 by Gessow and Tapscott and TN 3482 by Tapscott and Gessow.)

2. Amer, Kenneth B., and Gustafson, F. B.: Charts for Estimation of Longitudinal-Stability Derivatives for a Helicopter Rotor in Forward Flight. NACA TN 2309, 1951.

TABLE I

ITERATIONS FOR f = 12 SQUARE FEET

Iteration	α, deg	a', deg	λ	C_T	μ	$C_{P,i}$	$C_{P,p}$	$C_{P,c}$	$C_{P,o}$	C_P	$\theta_{0.75}$, deg
1	0	0	0	0.00418	0.483	0.000018	0.000274	0	0.000635	0.000927	20.5
2	-30.0	23.7	-.282	.00385	.418	.000015	.000274	0	.000338	.000627	14.9
3	-23.7	14.9	-.186	.00409	.442	.000017	.000274	0	.000446	.000737	16.7
4	-26.0	18.3	-.218	.00400	.434	.000016	.000274	0	.000423	.000703	16.1
5	-24.9	16.5	-.202	.00403	.440	.000017	.000274	0	.000443	.000734	16.6
6	-25.5	17.4	-.210	.00402	.435	.000014	.000274	0	.000428	.000716	16.4
7	-25.8	18.0	-.213	.00400	.435	.000017	.000274	0	.000432	.000723	16.8

TABLE II

FINAL ITERATION FOR ALL f VALUES

f, sq ft	α, deg	a', deg	λ	C_T	μ	$C_{P,i}$	$C_{P,p}$	$C_{P,c}$	$C_{P,o}$	C_P	$\theta_{0.75}$, deg	P, hp	$\alpha_{(1.0)(270^\circ)}$	$\alpha_{(u_T=0.4)(270^\circ)}$
0	-12.3	12.4	-0.109	0.00407	0.472	0.000017	0	0	0.000283	0.000300	9.0	1,096	10.2	9.0
6	-18.8	15.3	-.163	.00404	.455	.000017	.000136	0	.000402	.000555	13.6	2,028	15.4	13.7
12	-25.8	18.0	-.213	.00400	.435	.000017	.000274	0	.000432	.000723	16.8	2,642	15.8	12.0
18	-29.9	18.2	-.244	.00405	.419	.000017	.000410	0	.000433	.000860	18.9	3,145	15.6	10.6
24	-31.7	16.7	-.259	.00414	.409	.000018	.000547	0	.000463	.001028	20.6	3,752	20.1	15.1

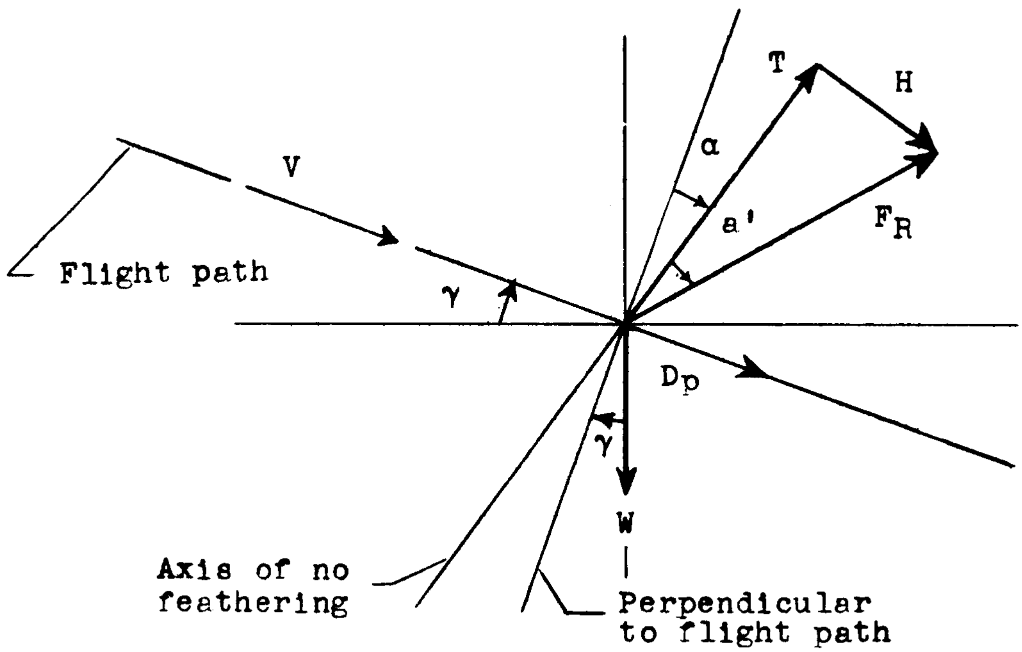

Figure 1.- Force and velocity diagram on a helicopter in climbing flight.

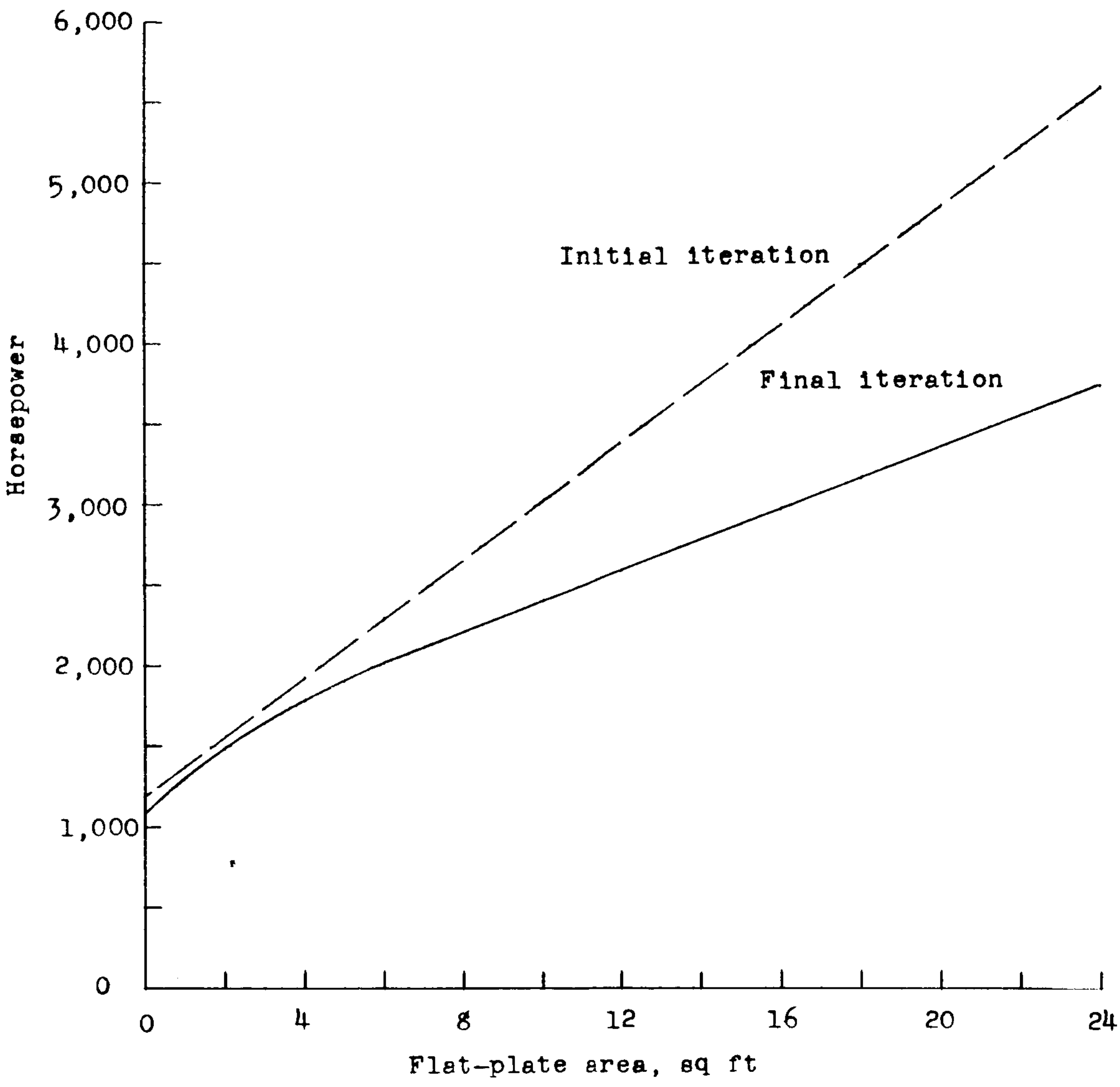

Figure 2.- Comparison of power-required calculations for a 12,000-pound, 200-knot helicopter. $\sigma = 0.10$; $\Omega R = 700$ fps; disk loading = 4.87 lb/sq ft; $\rho = 0.002378$; $\theta_1 = -8^{\circ}$.